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S-2636

M. Sc. - I (Sem. I) Examination

March / April – 2011

Mathematics

(Graph Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M.Sc. - 1 (sem. 1)**

Name of the Subject :

☛ **MATHEMATICS**

☛ Subject Code No. : **2 6 3 6** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Follow the usual notations and conventions.

(4) Figures on the **right** indicate full marks.

1 Attempt any **two** : **14**

(1) Discuss konigsberge bridge problem.

(2) Prove that in a connected graph G, any minimal set of edges containing atleast one branch of every spanning tree of G is a cut set.

(3) Prove that a connected graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exist no edge in G whose one end vertex is in subset V_1 and the other in subset V_2 .

2 Attempt any **two** : **14**

(1) Define component of a graph. Prove that a simple graph with n-vertices and K components can have at the most $\frac{(n-k)(n-k+1)}{2}$ *edges.*.

(2) Prove that the ring sum of any two cutsets in a graph is either a third cutset or an edge dis-joint union of cutsets.

(3) Define complete graph. Prove that in a complete graph with n-vertices there are $\left(\frac{n-1}{2}\right)$ edge disjoint hamiltonian circuits if n is odd number greater than or equal to three.

3 Attempt any **two** : **14**

(1) Define ring sum of two graphs. In a graph G, let P_1, P_2 be two different paths between two given vertices, prove that $P_1 \oplus P_2$ will be a circuit or a set of circuits.

(2) Prove that every circuit has an even number of edges in common with any cutset.

(3) Define minimally connected graph. Prove that a graph is a tree if and only if it is minimally connected.

4 Attempt any **two** : **14**

(1) Define height of a binary tree. Prove that the maximum possible height of an n-vertex binary tree is $\lceil \log_2(n+1)-1 \rceil$.

(2) Define incidence matrix. Prove that two graphs G_1 and G_2 are isomorphic if and only if their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns.

(3) Prove that every tree has either one or two centres.

5 Attempt any **two** : **14**

(1) Using Euler's formula, Prove that $K_{3,3}$ is non-planar.

(2) If A(G) is an incidence matrix of a connected graph with n-vertices prove that the rank of A(G) is (n-1).

(3) Define spanning tree. Prove that every connected graph has atleast one spanning tree.